HERIOT WATT UNIVERSITY

The Maths Newsletter

Feb 2023

@HeriotWattMathsNewsletter

maths-newsletter@hw.ac.uk

"Without Mathematics, There Is No Art"

By Miraal Sarki, Shitikshu Vyas, Clara Flegel

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By Antoine Goldsborough

With Support from the Jack Carr Fund

Department News and Events

Maths Café Talks

The Maths Café and the Maths Gym are very happy to introduce short talks on different mathematical topics aimed at an undergraduate audience. Recordings of previous talks can be found here https://mathsgym.hw.ac.uk/past-maths-cafe-talks/.

For more talks you can go to the Maxwell Institute website as they have year-round seminars that go over every part of mathematics. One talk from a member of staff from Heriot Watt is Wed 29th of March (2-3pm): Carmen Boado-Penas. For more seminars about interesting parts of maths check out https://www.maxwell.ac.uk/events/upcoming-events/.

The National Robotarium

In December of 2022 the state of the art National Robotarium was built, costing £22.4m, and this will allow research to advance on a global scale thanks to the dedicated researchers from Heriot Watt and the University of Edinburgh. Keep an eye on their website for interesting developments in AI; https://thenationalrobotarium.com/.

Scottish Uni Maths Olympiad (Sumo)

Students across Scotland, in groups of 2-4, will compete in a challenging event inspired by the British Mathematics Olympiad and Imperial College Maths Competition! For any further questions contact maths.society@hw.ac.uk.

Maths Society

Lucy Welford

We are a society open to all students with any sort of interest in Mathematics - Maths students, students opting to study a Maths elective, or those who enjoy Maths. We hope to organise a number of social events to meet those with the same Mathematical interests, as well as other academic and career related opportunities. Most importantly, we want to be a space where Maths students can connect, socialise and have fun!

School Officer

Jake Douglas

Jake is the new maths news officer. If at any point you'd like to give feedback on how your studies are going, need help with anything, or are struggling academically, please send me an email at so.maths@hw.ac.uk.

Maths Gym and Maths Café

Emma Coutts

The **Maths Gym** is a cross-campus initiative aimed at supporting all HW students, from **any** subject, to strengthen their mathematical or statistical skills and gain confidence in applying these skills. We provide support through a variety of activities including drop-in sessions, one-to-one appointments, and workshops.

Whether you want to brush up on basic skills or need help to understand new material from your course, don't hesitate to get in touch! We can help you achieve your maths or stats based goals, whatever they are. For more details please visit our website www.mathsgym.hw.ac.uk

The **Maths Café** is a peer supported initiative that allows maths students at Heriot-Watt to seek help among each other, ask questions or just chat with their peers, you can check out our website https://mathsgym.hw.ac.uk/maths-cafe/

Piscopia Initiative

Gemma Crowe

The Piscopia Society is a UK-wide initiative which aims to encourage women and non-binary students to pursue a PhD in Mathematics. To find out more about the society and events, such as our monthly PiWORKS seminar, please visit https://piscopia.co.uk/ or contact your Heriot-Watt representative: ggc2000@hw.ac.uk

The National Student Survey

The NSS is an independent UK survey, carried out annually, which provides final-year undergraduate students with an opportunity to reflect on their experience at university. If you are eligible to take part, you will receive an email from Ipsos MORI inviting you to complete the survey online. The NSS is open until the end of April 2023. Final year students can go directly to www.thestudentsurvey.com.

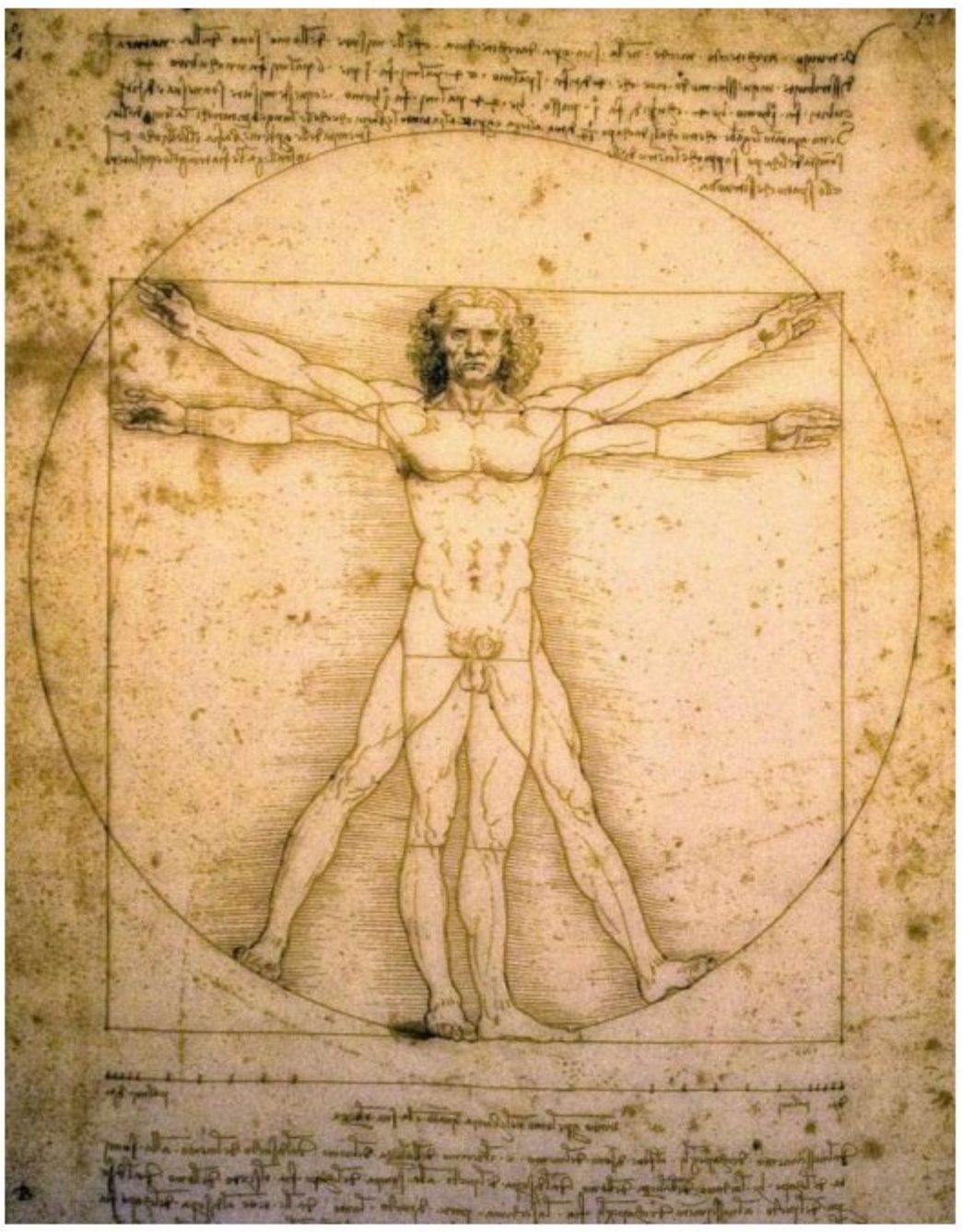
"Without Mathematics, There Is No Art"

-Luca Pacioli

By Miraal Sarki, Shitikshu Vyas, Clara Flegel

The quote above belongs to the Italian mathematician, Luca Pacioli. His birthplace, Tuscany, is home to some of the world's most famous art and architecture. Nicknamed as the 'father of accounting and bookkeeping', Pacioli collaborated and lived with the world-renowned artist, Leonardo da Vinci.

The pair worked together synergically; Pacioli taught Euclidean geometry to da Vinci, who in turn impressed Pacioli by using this in his mesmerizing artwork. *Divina Proportione*, a book on the golden ratio written by Luca, has numerous illustrations by his dear friend, da Vinci. Only two copies of the elusive book survive now! [2]
Amongst many other illustrations, the world-famous *Vitruvian Man* by Leonardo da Vinci appeared in this book and is still very respected to this day.



(Vitruvian Man, 1490)

Leonardo was inspired to produce this masterpiece by the works of a Roman architect named Vitruvius, who drew on the traditions of geometric symbolism to develop a theory of cosmic order and its applications to building projects.

While it's true that there are two pairs of arms and legs in the Vitruvian man, it's actually a combination of 16 possible poses! The circle on the outside depicts the cosmic and the divine due to its symmetry, while the square represents the earthly and the secular.

Leonardo described the proportions in the notes below the image; if you're having a hard time reading them, you're not alone! Leonardo had a peculiar way of writing; he would very often write in backward mirrored text. This was a way of protecting himself in case he wrote something controversial that angered the Church.

Although the symbolism of the Vitruvian Man is very fascinating, the integration of art and mathematics goes way beyond pictures and illustrations, as can be seen in the textile industry too!

Textile Mathematics

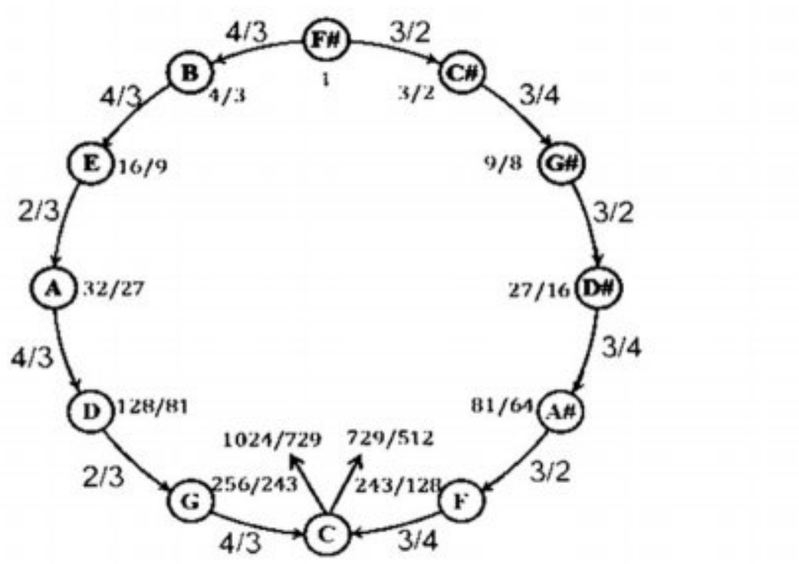
It may not be that intuitive, but indeed textiles involve the use of mathematics in their production. Geometry and number theory are used in mat weaving, while mathematical modelling is used in 'tiling of the plane', to ensure that the desired patterns are knitted. Knitting also uses, topology, symmetry or the number pi to ensure precision in production.[3]

(Knitting Patterns)

Moreover, computing colour matching is used to obtain the desired shades of colours, while also minimizing the cost of the process. Partial differentiation is used too, with the aim of predicting the alignment of dyes and saturation limit.

Pythagorean Tuning

Now moving on to another artistic field, we look at how the famous Greek mathematician, Pythagoras, formulated a tuning method for musical instruments. The frequency ratio intervals of the chords are in the ratio 3:2, also known as the perfect fifth. Just like the octave, this is another consonant, which put simply, is a pleasing sound to hear! In this, the notes alter by multiplying the previous note length by 3/2, and then making some relevant adjustments.



(Illustration of Proportions in Pythagorean Tuning)

M.C. Escher – The Mathematical Artist

M.C. Escher was a Dutch graphic artist known for his use of mathematical concepts in his art. His work explores the relationships between infinity, the structure of space, and the nature of reality. He was obsessed with capturing infinity into a finite space.

His art is clearly geometrically inspired, but also depicts his fascination with showing more abstract mathematical concepts - like symmetry, reflection, and recursion - in a visual manner. In fact, he once confessed that he preferred geometry to arithmetic and algebra because it appealed to his imagination!
He was also fascinated by the properties of the Möbius Strip, which is a one-sided, two-dimensional surface. This interest can certainly be seen reflected in his woodcuts, where he used the physical properties of wood, as well as light and shadow, to create the illusion of three-dimensionality, and enhance the depth and volume of his works.

He used impossible constructions (e.g., 'Ascending and Descending, 1960) and tessellations (an arrangement of shapes fitted together in a repeating pattern without gaps or overlaps) to create works that challenge the viewer's perception of reality.

(Ascending and Descending, 1960)

Escher famously made use of the 17-plane symmetry group (also called the 'wallpaper group') in his tessellation artwork, which he became familiar with through his friend, mathematician George Polya. The 17-plane symmetry group refers to the 17 distinct types of symmetrical patterns that can be found on a two-dimensional plane. It includes rotations, reflections, translations, and glide reflections, and each symmetry type is defined by specific properties, such as its number of rotational axes.
Arguably one of Escher's most famous tessellation works is 'Lizard', - interestingly, his repeating artworks notably featured aspects of nature, such as birds, horses, and fish! Escher was consistently blurring the line between sciences and art, showing the interplay between them and how one cannot exist without the other.

Polyclitus' Canon of Proportion – The Divine Ideal

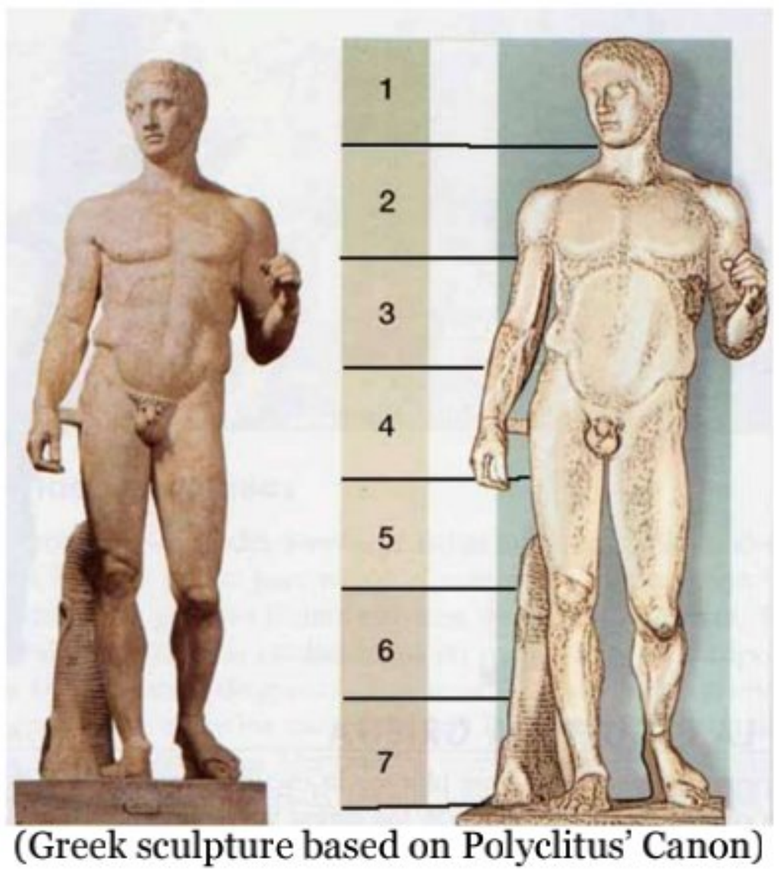
Another art form that is heavily based on mathematical properties is sculpture. The Greek sculptor Polyclitus was the first to assign the 'ideal' human body proportions using quantifiable values, in what he called 'The Canon'. It is essentially a set of proportions for the human body based on mathematical ratios, such as the golden ratio. [1]

According to the Canon, the height of the ideal human figure should be about eight heads, and each part of the body was determined to be in proportion to the whole. For example, the length of the outstretched arms was said to equal the height of the figure, and the length of the legs was said to equal four heads.

(Lizard, 1942)

This system of proportions was used by sculptors to create works that were considered aesthetically pleasing and harmonious. The use of the Canon ensured that the sculptures produced during this period had a consistent and recognizable style, and it became an important part of the classical Greek artistic tradition.

In modern times, digital technologies such as 3D modelling software and computer-controlled machinery have made it easier for sculptors to use mathematical concepts in their work. For example, a sculptor can use a computer to model a sculpture and test different proportions and shapes, before physically constructing the final work.



(Greek sculpture based on Polyclitus' Canon)

References

[1] M. Guía, *The Canon of Proportions in Ancient Greek Sculpture: Searching for a Divine Ideal*, Artsper Magazine, 2022

[2] Daryl Green, *Luca and Leonardo- The Divine Proportion and a life-long Renaissance friendship, March 2018*

[3] Dr. Kanikicherla Rani, *Pramana Research Journal, Volume 8, Issue 12, 2018*

Who Killed Evariste Galois?

By Antoine Goldsborough

The name 'Galois' is often associated to both his genius and the circumstances of his death. What I have always found incredible is that by the age of 20 he had developed a theory (now known as *'Galois theory'*), which is nowadays usually taught as a fourth- or fifth-year undergraduate course. When students learn about this beautiful theory, they are only one or two years older than Galois was when he made these discoveries.
In this article, we will glance at his main contribution to mathematics, dive into 19th century political France, and look at the speculations surrounding his death.

Galois' most famous contribution to mathematics was proving that there is no formula for solving a quintic equation and he, more than any other mathematician, realised why this was the case. Without going into excessive detail: Galois realised that, in order to understand whether a solution by radicals existed for a given equation, one should look at the collection of permutations that preserve the equation. This group of permutations, called the '*Galois group*' of the equation, then needs to be *solvable* for the original equation to be solvable by radicals. He showed that, for all n>4, there is an equation of degree n that is not solvable by radicals. Among other things, he introduced a systematic way of testing whether an equation is solvable by radicals, and Galois was also the first to use the term '*group*'. His ideas were notoriously difficult for his contemporaries to understand and hence remained obscure outside of France for a long time.

At the time of these mathematical discoveries, France was in political turmoil - in July 1830, the 'Trois Glorieuses' ('Three Glorious Days') revolution broke out. These events were considered the second French Revolution, and ultimately saw Louis-Philippe I succeed his cousin Charles X as King of the French. During this time of political instability, Évariste Galois joined the Republican artillery unit of the national guard. Whilst at the École Normale where he was studying, he also wrote articles criticizing the way the sciences were being taught to students. According to him, the education system at the time was merely showing students how to reproduce what they had learnt, rather than teaching them to think for themselves. These articles, along with the fact that he was a staunch republican, led to his expulsion from the École Normale in 1831.

Later that year, at a banquet with fellow republicans, Galois proposed a toast 'to Louis-Philippe' while brandishing a dagger. This was interpreted as a threat to the King, so he was arrested and sent to the Sainte-Pélagie prison, where he was later acquitted. On the day following his release, Galois joined a protest wearing the Republican artillery uniform and was heavily armed with pistols, a loaded rifle and a dagger. He was consequently arrested once more and sentenced to six months in prison, to be released in April 1832.
The infamous duel which led to Galois' demise took place on 30 May 1832. Likely, this duel was about a woman ('Je meurs victime d'une infâme coquette') - five days prior to the duel, he wrote a letter to his friend Auguste Chevalier in which he mentioned a broken love affair.
He stayed up all night before the duel writing letters to his republican friends, including one for Chevalier. This letter has become incredibly famous and is considered his mathematical testament, in which he explains some of his main ideas. Hermann Weyl stated, "This letter, if judged by the novelty and profundity of ideas it contains, is perhaps the most substantial piece of writing in the whole literature of mankind." In the margins of this letter, one can see several phrases, like "Je n'ai pas le temps" ("I don't have time"). In the early hours of May 30th, he got shot in the abdomen and died the following day at the Cochin hospital.

There was much speculation surrounding the identity of Galois' duellist. Considering the political events that were taking place in France at the time, and Galois' radical republican views, has led some biographers such as Eric Bell (in his book *Men of Mathematics)* to interpolate that his death was orchestrated by the police and royalists in order to eliminate a political enemy. Even Évariste Galois' brother, Alfred, was convinced of a political conspiracy.

This is one of many hypotheses about his death, although based on very scarce historical evidence. One of Galois' friends, writer Alexandre Dumas (author of the *Three Musketeers*) names Pescheux d'Herbinville as Galois' opponent, however there is no evidence that this was the person who killed Evariste. Newspaper articles from the time give a description of the adversary as being one of Galois' friends, probably Ernest Duchatelet, who had been imprisoned at the same time in Sainte-Pélagie prison.

As for the name of the woman who Galois was pursuing, historical investigations suggest that this woman was 'Stéphanie-Félicie Poterin du Motel'. She was the daughter of the owner of the hotel where Galois spent his last few months. Her name appears in the letters written prior to his death. Galois' last words to his brother were "Don't weep, Alfred! I need all my courage to die at twenty!".

The mystery surrounding Galois' death might never be elucidated. What is certain is that, by his incredible mathematical genius, he paved the way for the birth of modern algebra and group theory.